Modeling Soil Matrix Hydraulic Properties for Variably-Saturated Hydrologic Analysis

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Abstract: This article presents a method to estimate soil matrix hydraulic properties to be used for variably-saturated hydrologic analysis, such as rainfall/runoff and stormwater management analyses, among others. The methods are described and verified with the U.S. Department of Agriculture's Unsaturated Soil Database (UNSODA), and the resultant soil hydraulic properties are presented. For conservative estimates of soil infiltration, a filter is recommended to remove soils from the analysis with high-saturated hydraulic conductivity that are likely to be macropore-dominated, and it is confirmed that the removed soils contain substantially more macropore influence than the retained soils. It is shown with the UNSODA database that good estimations of the soil matrix hydraulic conductivity function may be obtained by using the modified van Genuchten-Mualem model normalized to the saturated hydraulic conductivity with an immutable model parameter $h_e = -0.1$ cm if data are not available to calibrate h_e; this allows for the hydraulic conductivity curve to be estimated without laboratory-measured unsaturated data and overcomes some of the limitations of the van Genuchten-Mualem model. **DOI: 10.1061/JSWBAY.0000808.**

Introduction

Next to rainfall, soil hydraulic properties (SHP) are the foundation of hydrologic analysis. Due to the difficulty and cost of obtaining appropriate SHP in the laboratory, textural classes are often used as a proxy. For example, in the case of infiltration stormwater control measures, site selection, type, and performance expectations are often mandated based on the soil textural class at the site. However, such decisions fail to recognize that within-class variability of SHP may be as large as or larger than between-class variability, indicating that textural class proxies are not sufficient in many situations for predicting the hydraulic behavior of soils (e.g., Gutmann and Small 2005; Loosvelt et al. 2011). To overcome the limitations of textural-class proxies, there is a need for SHP models that describe the expected behavior of individual soils, which are often needed to predict the performance of rainfall/runoff and stormwater management practices. Such models could be used as inputs in Richards equation solvers such as Soil Water Atmosphere Plant (SWAP) (van Dam and Feddes 2000; Kroes et al. 2009). The use of these SHP models, as opposed to traditional class-average values, will improve the ability of engineers to predict the performance of watersheds and stormwater controls, among other projects.

Several hydrologic analyses have been performed recently in various research areas using the hydraulic properties of individual soils rather than class-average properties (e.g., Gutmann and Small 2005; Twarakavi et al. 2009, 2010; Loosvelt et al. 2011; Coppola et al. 2014; Hohenbrink and Lischeid 2014). Many of these studies have used pedotransfer functions (PTF) to estimate SHP based on soil characteristics such as grain-size distribution; however PTF do not fully replicate within-class variation of SHP, and may also contain significant estimation errors (Tietje and Hennings 1996; Weynants et al. 2009; Vereecken et al. 2010; Loosvelt et al. 2011). While PTF are popular due to their use of easily-obtained laboratory values when compared to laboratory-obtained SHP data, the use of measured soil properties would be superior as long as sufficient data are available.

In addition to the need to simulate individual (rather than class-average) soils, commonly applied techniques for modeling SHP have several limitations that affect their usefulness for designers. The most widespread SHP model is the van Genuchten-Mualem model (VGM) (van Genuchten 1980), which is limited in the following three ways: (1) it does not properly model the hydraulic conductivity curve when the fit-parameter n is close to 1.0 (Vogel et al. 2000; Ippisch et al. 2006), which tends to be the case for fine-textured soils; (2) it does not have a well-defined transition between saturated and unsaturated flow (Schaap and van Genuchten 2006; Børgesen et al. 2006) which makes it difficult to parameterize for variably-saturated analyses (Vereecken et al. 2010); and (3) it requires difficult-to-obtain unsaturated hydraulic conductivity data to properly estimate the hydraulic conductivity curve (Schaap and Leij 2000). Furthermore, engineers often use SHP to estimate runoff from watersheds and design stormwater controls that rely on infiltration. In such cases, SHP estimated from measurements of undisturbed soil samples that contain macropores introduce a substantial non-conservatism into the design process. Because the macropore structure of a watershed or stormwater control site generally cannot be known a priori, using the SHP of only the soil matrix (that is, the soil in the absence of macropores) is a conservative design approach. Widely available estimates of SHP, however, typically fail to distinguish between matrix-dominated and macropore-dominated soils.

These are fundamental issues in soil physics and there is not presently a complete solution; however, this article presents a discussion of these problems along with a particular solution so that a variably-saturated hydraulic model of the soil matrix may

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be obtained from soil hydraulic measurements. The method presented in this article consists of using the Rawls et al. (1998) soil-matrix conductivity data to filter out soils that would be representative of macropore-dominated flow rather than soil-matrix flow, followed by an application of the modified van Genuchten-Mualem model (MVGM) (Ippisch et al. 2006), which rectifies the poor fit of the VGM model for soils with a value of n close to 1.0. By combining these two steps, it will be shown that the MVGM hydraulic conductivity model may be normalized to the saturated hydraulic conductivity (K_s) rather than an arbitrary conductivity value (K_0), ensuring continuity of hydraulic behavior in variably-saturated flow simulations. The U.S. Department of Agriculture's Unsaturated Soil Database (UNSODA) containing soil hydraulic measurements (Liej et al. 1996; Nemes et al. 2001) is used as a verification of the methods and to calibrate the MVGM model parameter h_e. Typical SHP model parameter estimates generated from the UNSODA database are presented.

Methods

Soil Hydraulic Property Model

Soil hydraulic property models consist of two equations: the soil/water characteristic curve (SWCC; also known as the water retention function) and the hydraulic conductivity curve (HCC; also known as the permeability function). The most widespread of these models is the VGM model [Eqs. (1*a*) and (1*b*)], which is popular due to its goodness of fit to many data sets (Vereecken et al. 2010) and also because the HCC has a closed-form solution that is easy to handle in numerical simulations. The VGM model is given by [Eqs. (1*a*) and (1*b*)] which are the SWCC and HCC, respectively

$$S_e = \frac{\theta - \theta_r}{\theta_s - \theta_r} = \begin{cases} [1 + (\alpha|h|)^n]^{-m} & h < 0 \\ 1 & h \geq 0 \end{cases} \tag{1a}$$

$$K_r = \begin{cases} S_e^{\tau} \cdot [1 - (1 - S_e^{1/m})^m]^2 & S_e < 1 \\ 1 & S_e \geq 1 \end{cases} \tag{1b}$$

where S_e = effective degree of saturation [dimensionless, 0–1]; θ = water content [L/L, θ_r to θ_s]; θ_s and θ_r = saturated and residual water contents, respectively; α = a model fit parameter [1/L]; h = pressure head [L, negative in suction]; n = model fit parameter [dimensionless, greater than 1]; $m = 1 - 1/n$; K_r = relative hydraulic conductivity [dimensionless, 0–1]; and τ = a model fit parameter [dimensionless]. Based on Mualem (1976), τ is often assumed to be 0.5. The relative hydraulic conductivity K_r is proportional to the conductivity K by a normalization constant K_0 [Eq. (2)]. The remaining free parameters θ_s, θ_r, α, and n are typically estimated by fitting water retention data. The saturated water content is close, but usually less than, the porosity of the soil because under normal saturating conditions some isolated pockets of air will remain in the soil

$$K = K_0 K_r \tag{2}$$

Schaap and Leij (2000) use the VGM model by normalizing the HCC to a fitting parameter K_0 which is less than K_s. When K_0 is less than K_s, it is being assumed that conductivity near saturation is dominated by cracks and macropores (Minasny et al. 2004), whereas the fully unsaturated conductivity is a result of the soil matrix. Therefore only the unsaturated soil matrix flow is estimated, so this VGM model is generally only applicable for matric suction greater than 3 cm (Vereecken et al. 2010). However, use of the Schaap and Leij (2000) model requires unsaturated conductivity data to estimate K_0 and only applies to saturated flow if K_0 is equal to K_s. The VGM model has evolved such that K_0 and τ are not considered physical constants at all but are purely considered fitting parameters, and even m has become an additional fitting parameter (Schaap and Leij 2000; Vereecken et al. 2010), resulting in the need for water content and conductivity data over a range of suctions to fit the model.

In addition to the need for HCC data to estimate K_0 and the lack of correspondence to K_s, perhaps the largest drawback of the VGM model is the unrealistic HCC generated when the parameter n is small (less than approximately 1.2 or 1.3). When n is that small, the VGM model HCC contains a precipitous drop in conductivity near saturation; this occurs at suctions smaller than any reasonable air-entry suction [which should not be less than 0.58 cm according to Ippisch et al. (2006)] which confirms that this drop is not a physical phenomenon but rather an artifact of the mathematical model (Ippisch et al. 2006). Vogel et al. (2000) present a modification to the VGM model that rectifies the precipitous drop in conductivity near saturation by adding an additional fitting parameter. Schaap and van Genuchten (2006) and Ippisch et al. (2006) expand on this work and relate the fitting parameter to the air-entry pressure, although they note that the parameter may not be physically the same as the air-entry pressure. Ippisch et al. (2006) showed that failure to modify the VGM model for low-n soils may result in unrealistic results in infiltration simulations due to the unrealistic HCC, along with numerical convergence problems. Because fine-textured soil classes tend to have n close to 1, hydrologic analyses performed with the standard VGM model may have heavily biased results for fine-textured soil classes; indeed it has been shown that n is the most-sensitive parameter in hydrologic simulations when using VGM (Gutmann and Small 2005; Rocha et al. 2006).

For the methods presented in this study, the SHP are computed using the modified van Genuchten-Mualem (MVGM) model (Eqs. 3*a* and 3*b*) as presented by Ippisch et al. (2006). Schaap and van Genuchten (2006), and Børgesen et al. (2006) further modify the MVGM model by introducing an empirical correction equation to the hydraulic conductivity near saturation. However, in the method presented in this paper, the relative hydraulic conductivity will be normalized to K_s [Eq. (4)] making such corrections unnecessary. This allows for coupled saturated/unsaturated flow simulations that can occur in hydrologic analyses. However, if the macropore-dominated samples are not first removed from the dataset, this method of normalizing the HCC to K_s could lead to data that is skewed to the high-conductivity range. The MVGM model is given by

$$S_e = \frac{\theta - \theta_r}{\theta_s - \theta_r} = \begin{cases} \frac{1}{S_c} \cdot [1 + (\alpha|h|)^n]^{-m} & h < h_e \\ 1 & h \geq h_e \end{cases} \tag{3a}$$

$$K_r = \begin{cases} S_e^{\tau} \cdot \left[\frac{1-(1-(S_e S_c)^{1/m})^m}{1-(1-S_c^{1/m})^m}\right]^2 & S_e < 1 \\ 1 & S_e \geq 1 \end{cases} \tag{3b}$$

where $S_c = [1 + (\alpha|h_e|)^n]^{-m}$; and h_e (L, less than zero indicating suction) = model parameter related to the air-entry pressure. The other parameters are treated in the same way as the standard VGM model. While h_e is related to the air-entry pressure, they do not have to be equal. Mathematically, $|h_e|$ must be less than or equal to the air-entry suction; in many cases a small $|h_e|$ will not alter the shape of the HCC, while in other cases h_e may approximately represent the air-entry pressure, depending on the values of the other parameters. While the VGM model typically assumes a normalization parameter $K_0 < K_s$ for the conductivity equation (Eq. 2), this proposed method will use the measured K_s (Eq. 4) to simplify the conductivity predictions and ensure continuity of the conductivity equation in coupled saturated/unsaturated flow simulations.

$$K = K_s K_r \tag{4}$$

While K_s may be used as the conductivity normalization constant, introducing h_e as a fitting parameter still results in the need for unsaturated HCC data, which is why the better-fitting MVGM model was avoided by Weynants et al. (2009). To avoid this necessity and to have a generalized method for predicting the HCC based on the SWCC and K_s only, this study will explore h_e as a MVGM model constant: immutable for all soils rather than varied to fit each individual soil. The model fit will be verified with soils from the UNSODA database that have unsaturated HCC data, allowing for an assessment of the effect of h_e and for estimating its optimal value when unsaturated data are not available and it is used as an immutable parameter.

Database Selection

The UNSODA soil database (Liej et al. 1996; Nemes et al. 2001) contains the soil hydraulic measurement data that will be used to verify the proposed methods and estimate the soil matrix hydraulic properties. This database is available to the general public and has been used in many published research studies. The UNSODA database contains a set of measurements for hundreds of individual soils, including data for water content, conductivity, textural class, density, etc., although not all soil entries have data entered for every parameter. The preprocessing of the database begins with selecting soils that have sufficient data to estimate the MVGM model parameter values; namely, a measured value for K_s and at least five data points on the SWCC (because there are four parameters in Eq. (1*a*), five points is the minimum number needed to avoid overfitting the data). For this study, only the laboratory drying data are used because there is insufficient wetting data to verify the proposed methods for a wide range of soils. Therefore, this study is limited in that it does not address wetting/drying cycle hysteresis; however, the primary goal of these methods is the development of SHP for variably-saturated flow conditions, where the unsaturated soil properties are often of secondary significance compared to the saturated properties (Fredlund 2006). Further, soils where the textural class was not provided in the database are excluded from this study. One soil was removed due to insufficient data in the low-suction range, and 10 water-content data points were removed as obvious outliers from visual inspection of the data (see Appendix for more details). This preprocessing yields 374 soils. The UNSODA database provides separate entries for θ_s and the porosity; it was assumed for this analysis that when only the porosity is available in the database the two terms are equivalent, otherwise θ_s was used directly for fitting the SWCC.

Filtering to Remove Macropore-Dominated Soils

Macropore structures exist in surface soils and may significantly increase the saturated flow rates even at the field scale. However, detailed knowledge of their structure is not generally possible; Weynants et al. (2009) also conclude that for large-scale studies, it is reasonable to exclude highly localized effects (such as macropores). Because of this uncertainty, the method presented in this study filters out the macropore-dominated soils; this has the effect of being conservative for many hydrologic simulations because macropores can only increase the infiltration beyond the soil matrix (and thus, reduce runoff). In many fields such as the design of stormwater controls, this conservatism is an appropriate design constraint.

Macropores affect the flow only from saturation (zero suction) up to about 10 cm of suction (Jarvis 2007). Therefore, it can be concluded that the MVGM model (or any other SHP model) would overpredict the HCC at suctions greater than 10 cm in the macropore-dominated soils if the relative conductivity function is normalized to K_s. To illustrate this concept, Figs. 1(a and b) show two soils that are evidently macropore-dominated, as seen when the models and unsaturated conductivity measurements are normalized to K_s as in Eq. (4): (a) a clay soil with $K_s = 1{,}668$ cm/d, and (b) a silty clay loam soil with $K_s = 168$ cm/d. These two soils appear macropore-dominated because the unsaturated HCC data trend toward a $K_r \ll 1$ until the suction approaches zero. As discussed previously, the hydraulic conductivity should not be able to drop significantly until the suction is greater than the air-entry suction, but in the two soils in Figs. 1(a and b), the saturated hydraulic conductivity is much higher than the conductivity near the air-entry suction (which appears to be in the range of 10^1 or 10^2 cm based on the data). In the standard VGM method described by Schaap and Leij (2000), these soils would be normalized to a $K_0 \ll K_s$ by fitting the HCC data [Eq. (2)], likely resulting in better HCC fits

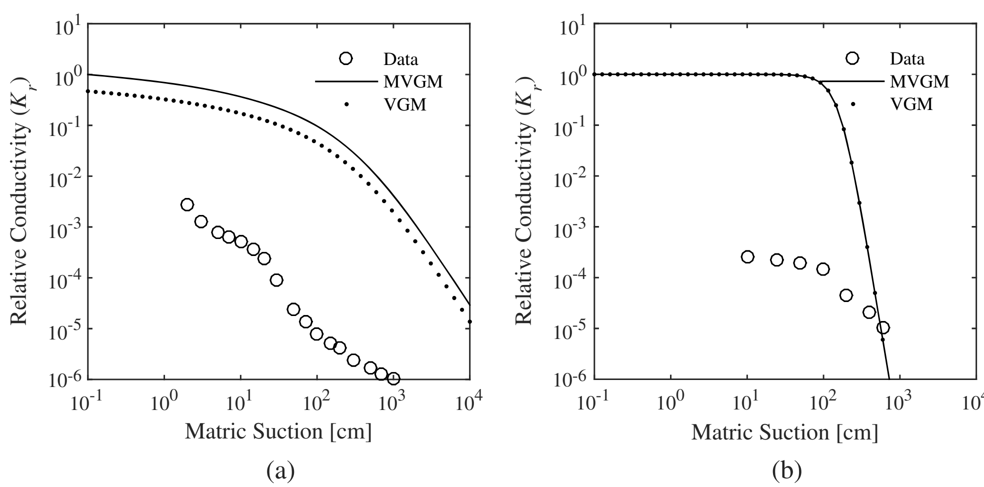


Fig. 1. Influence of macropores is evident in these two soils because the HCC models are normalized to the saturated hydraulic conductivity; the models substantially overpredict conductivity in the low-suction range

in the unsaturated range as opposed to these poor fits using Eq. (4). However, the K_0 fit ignores K_s, making it useful only when unsaturated conductivity data are available and when it is not necessary to fit K_s for saturated flow.

Rawls et al. (1998) present the statistics of expected soil-matrix conductivity by soil textural class and report that they eliminated samples with K_s "greater than one order of magnitude different from the values reported by Rawls et al. (1982) because it was thought that [the soils with] large K_s values contained macropores and were not representative of the soil matrix." The Rawls et al. (1998) dataset contains 953 soil samples after removing suspected macropore samples. With such a high number of samples, Rawls et al. (1998) believe that the results are indicative of the soil-matrix conductivities that may be expected among and within the soil textural classes. Therefore, the Rawls et al. (1998) dataset is used in this study as an estimate of the expected range of K_s that would be observed in each soil textural class without macropore influence. An upper limit of K_s will be proposed based on the Rawls et al. (1998) data such that soils from other sources (such as UNSODA) with K_s below the upper limit may be assumed to be matrix-dominated because they are from the same population as the Rawls et al. (1998) soils.

Rawls et al. (1998) present a table of geometric mean and 25 and 75% (Q1 and Q3) values for K_s for 11 soil textural classes, excluding silt. They also present values for the fine sand, fine loamy sand, and fine sandy loam subclasses. To simplify the data and approximate one distribution of K_s for each textural class, this study averaged these subclasses into the respective class using a weighted geometric average. The resulting approximate textural class distributions are presented in Table 1. Assuming K_s approximately follows a lognormal distribution in each soil class (Tietje and Hennings 1996; Schaap and Leij 2000), the interquartile range (IQR) should be analyzed in log space as $\log_{10}(Q3) - \log_{10}(Q1)$. Observations outside the range of $[Q1 - 1.5 \cdot IQR, Q3 + 1.5 \cdot IQR]$ are often considered outliers; in a normal distribution, this range represents over 99% of the data. However, in the UNSODA database, $Q3 + 1.5 \cdot IQR$ is several orders of magnitude above the mean, putting it well above the Rawls et al. (1998) data cutoff. Therefore an upper limit that is smaller than $Q3 + 1.5 \cdot IQR$ is needed for filtering K_s to approximate the Rawls et al. (1998) data.

For the proposed method, the upper limit of K_s is set to $Q3 + 0.75 \cdot IQR$, as computed on the log-transformed data. This limit (which eliminates the upper 5% tail of a normal distribution) was selected because when applied to the UNSODA data, the resulting dataset closely matches the Rawls et al. (1998) soil-matrix data (Table 1). However, in the UNSODA dataset, this cutoff eliminates 21% of the samples (78 out of 374), highlighting the substantial presence of macropores in saturated conductivity measurements from undisturbed soil samples (about 90% of the UNSODA soils are undisturbed cores). While this K_s limit was selected somewhat arbitrarily, the removal of these soils will later be tested against the hypothesis that most soils with K_s above this limit are heavily influenced by macropores. It is highly unlikely that any K_s cutoff limit would perfectly filter out macropore-dominated samples; not only do macropores affect the conductivity in varying degrees, but other factors such as compaction may affect K_s as well (Pitt et al. 2008). For these reasons, the authors believe that the selected cutoff limit is an appropriate value to use for practical research as long as this limit is confirmed to remove macropore dominated soils on average, and as long as it is understood that the true range of expected soil properties is only being approximated. Table 1 shows the number of samples by class along with the geometric mean K_s from UNSODA before and after removal of the samples suspected to be macropore-dominated, compared to the Rawls et al. (1998) data. It is clear from the summary that many of the removed samples had conductivities that substantially skewed the group means. For each class, the K_s filter appears to move the UNSODA K_s distributions closer to the Rawls et al. (1998) distributions, although it is not possible to test for statistical significance between the groups.

Soil Hydraulic Property Model Parameter Estimation

The filtered UNSODA dataset containing measurements from 296 soils was used to estimate the parameters for the MVGM model [Eq. (2)]. For each individual soil, the HCC was normalized to that soil's measured K_s [Eq. (4)], but the parameter h_e is being considered immutable; therefore the optimum value of this parameter must be found. While h_e can and should be fitted to each individual soil when unsaturated conductivity data are available to estimate its value, here it is being considered immutable so that estimates for SHP models can be obtained when unsaturated conductivity data are not available. To calibrate the model parameters, the SWCC data were individually fitted with a least-squares regression in θ versus h, using the *MATLAB* function lsqnonlin, which estimates the best-fit parameters θ_s, θ_r, α, and n. The average root-mean-square (RMS) residual for θ is less than 0.01 cm/cm, which

Table 1. Saturated Hydraulic Conductivity Statistics

	Rawls et al. (1998)		UNSODA		UNSODA filtered		
Texture	N	K_s (cm/d)	N	K_s (cm/d)	K_s Upper limit (cm/d)	N	K_s (cm/d)
Clay	41	4.6 (1.7, 16.3)	22	32 (3.2, 106.6)	90	15	6.4 (1.6, 51.4)
Clay loam	73	2.9 (0.7, 16.8)	9	115.8 (13, 501.1)	178	4	6.6 (4.4, 18.7)
Loam	109	12.5 (6.7, 48.2)	33	54.4 (14.9, 233.3)	212	24	23.2 (10.9, 68.2)
Loamy sand	61	151.7 (86.7, 329.5)	33	212.9 (65.1, 558)	897	29	159.3 (60.4, 499)
Sand	92	316.4 (230, 602.1)	90	399.1 (155.8, 1360.3)	1,239	66	186.9 (86.7, 536.7)
Sandy clay	17	2.4 (0.7, 7.2)	2	0.1 (-)	40	2	0.1 (-)
Sandy clay loam	130	8.4 (2.4, 39.1)	18	6.5 (0.8, 30.7)	317	16	3.5 (0.6, 21.7)
Sandy loam	247	47.2 (20.7, 134.1)	54	47.9 (18.5, 144.9)	545	52	42.7 (18.2, 120.1)
Silt	0	-	2	55.8 (-)	-	2	55.8 (-)
Silt loam	107	18.5 (8.6, 55)	74	34.3 (10.6, 118.8)	220	62	19.6 (7.5, 58.6)
Silty clay	17	3.8 (1.2, 17)	15	42.8 (10.9, 200.5)	125	8	10.2 (3.1, 69.6)
Silty clay loam	59	10.3 (5.5, 31.2)	22	34.6 (11.7, 150.5)	114	16	17.8 (2.1, 92.8)

Note: Geometric mean and number of samples (N) for different datasets. Interquartile range (25, 75%) shown in parentheses.

is under the typical value of 0.017 cm/cm in data explored by Vereecken et al. (2010).

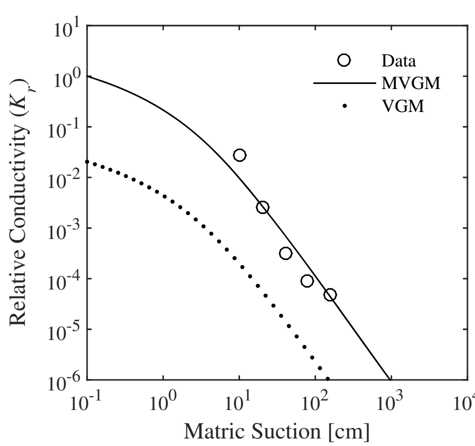
For the HCC, K_s was used directly from the filtered UNSODA database as the normalization constant [Eq. (4)] for each soil, and comparisons were made between the unsaturated HCC data and the MVGM model using a constant value of τ (0.5) and h_e for all soils, rather than considering these as fitting parameters. Since the HCC were not fitted to each individual soil's data, the comparisons are made to explore the optimum value of h_e for when it is used as an immutable model constant and to verify the MVGM model fit. However, because neither the VGM nor the MVGM models accurately represent the HCC at very high suctions (Fredlund 2006), the HCC comparisons will only be made if the measured K is greater than 0.01 cm/d to avoid these low-conductivity data skewing the verification results. This conductivity is an estimate of the field-capacity flux (Twarakavi et al. 2009) and so is assumed to be a reasonable lower limit of performance in many hydrologic simulations. However, neglecting the fit for very small conductivities is an assumption of this method, and so the results are limited to applications where flow rates are greater than 0.01 cm/d. Lower flow rates might be of interest for some long-duration vadose zone studies, but in such cases other methods of estimating the SHP may be needed.

Results and Discussion

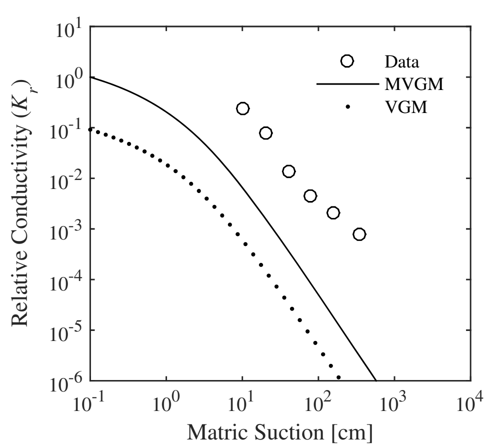
As an example to illustrate the effect of h_e along with some of the variability of the HCC data, Figs. 2(a and b) show the relative conductivity function for an example: (a) clay soil with $n = 1.04$, and (b) loam soil with $n = 1.1$, using $h_e = -0.1$ cm as an example (a small value for this parameter). The clay has the lowest n of the dataset, yet happens to be fitted optimally with this small value of $h_e = -0.1$ cm. The loam has a higher n, yet would be fitted better with a larger magnitude h_e. Due to the random nature of the error, some other soils might be better fitted with $h_e = 0$ or any other value. Fig. 2(b) was included to highlight that when using the immutable-h_e approach, not all soils will be optimally fitted in the unsaturated range, however the sacrifice of suboptimal fitting in the unsaturated range comes at the benefit of fitting the saturated conductivities exactly, and also the ability to estimate the HCC without unsaturated conductivity data. Figs. 2(a and b) also show that the conductivity is 1–2 orders of magnitude lower than the saturated conductivity when the matric suction is only 0.1 cm using the VGM model (the drop in relative conductivity from $K_r = 1.0$ at saturation to $K_r = 10^{-1}$ to 10^{-2} at 0.1 cm suction is not shown because of the logarithmic scale); the MVGM with $h_e = -0.1$ cm rectifies this problem by normalizing the relative conductivity to 1 when the suction equals $|h_e|$.

When introducing the MVGM model, Vogel et al. (2000) recommended a value of $h_e = -2$ cm when no other information is known about the soil because they were assuming that the air-entry pressure would be the best value to use for h_e, and -2 cm is a small air-entry pressure. Ippisch et al. (2006) used a minimum value of $h_e = -0.58$ cm based on a computed air-entry pressure for a large macropore, as they also supposed that h_e would be equivalent to the air-entry pressure and so smaller magnitudes would not be expected. Other studies typically place the expected value of air-entry pressure around -4 to -10 cm (Vereecken et al. 2010). While Ippisch et al. (2006) tried to relate h_e to the physical air-entry pressure, they also noted that h_e may not be the same as the physical air-entry pressure. The minimum values recommended be Vogel et al. (2000) and Ippisch et al. (2006) were tested for fits to this dataset, and based on the results of those fits a smaller value was also tested ($h_e = -0.1$ cm). Table 2 presents the statistics of mean residuals and RMS residuals in $\log_{10}(K)$ from the fits for all data with K greater than 0.01 cm/d (a negative mean residual indicates that the model is predicting lower K than the data, which is opposite to common regression residuals). From Table 2, it can be seen that when $n < 1.1$, the VGM model (which is equivalent to the MVGM model when $h_e = 0$) with the HCC normalized to K_s drastically underpredicts the unsaturated conductivity (evidenced by the mean residual equal to -1.51 orders of magnitude). This is consistent with the known limitation of the VGM model as shown by Vogel et al. (2000) and Ippisch et al. (2006).

Based on the results in Table 2, the best fits for h_e are -2 cm when $n < 1.1$, -0.1 cm when $1.1 \le n < 1.2$, and 0 otherwise. However, if a single, immutable value of h_e is adopted for all soils, the best fit is with $h_e = -0.1$ cm. The change from $h_e = 0$ to $h_e = -0.1$ cm results in a very large improvement in fit at low n (from a mean-log error of -1.51 to -0.53) and a negligible change at high n [from a mean-log error of 0.43 to 0.46; see Fig. 1(b) for a high-n soil that highlights the negligible difference between VGM and MVGM]. In the range $1.2 \le n < 1.3$, increasing $|h_e|$ moves the mean error away from zero; however, the change in mean and RMS error is small. Further increasing $|h_e|$ beyond 0.1 cm results in diminished returns at low n, and worsening of



(a)



(b)

Fig. 2. HCC plots highlighting that an immutable h_e will (a) sometimes fit well; (b) sometimes not

Table 2. Statistics of MVGM Model Residuals

	All Soils (175 soils)		$n \geq 1.3$ (87 soils)		$1.2 \leq n < 1.3$ (33 soils)		$1.1 \leq n < 1.2$ (41 soils)		$1.0 < n < 1.1$ (14 soils)	
h_e (cm)	Mean residual	RMS residual	Mean residual	RMS residual	Mean residual	RMS residual	Mean residual	RMS residual	Mean residual	RMS residual
0	0.06	1.01	0.43[a]	0.91	0.24	0.96	−0.34	1.07	−1.51[a]	1.58
−0.1	0.30[a]	0.93	0.46[a]	0.92	0.46[a]	0.98	0.13	0.96	−0.53[a]	0.84
−0.58	0.43[a]	0.93	0.51[a]	0.91	0.60[a]	1.02	0.34[a]	0.96	−0.23	0.73
−2	0.56[a]	0.96	0.56[a]	0.93	0.77[a]	1.09	0.56[a]	1.00	0.06	0.70

Note: Statistics of mean $\log_{10}(K)$ fit residuals between the MVGM model with various h_e and the filtered UNSODA data. These are statistics of mean residuals over each soil, for all $K_{\text{measured}} > 0.01$ cm/d. Residuals are grouped by the MVGM model parameter n. Negative mean residuals indicate that the model underpredicts the data.
[a]Means are significantly different from zero at 95% confidence level using Student's t-test.

fits at high n. The overall RMS residual on log-K with $h_e = -0.1$ cm is 0.93. This is not small, but is substantially better than models from a typical pedotransfer function (ranging from 1.24 to 3.01; Vereecken et al. 2010). If h_e were fitted to each individual soil, an improvement in the RMS residual would be expected: Weynants et al. (2009) report a RMS residual of 0.705 using the individually fitted MVGM model. However, using the individually fitted h_e would leave only the 175 soils from the UNSODA database that have unsaturated conductivity data; whereas using the immutable-h_e method allows for all 296 soils to be used that have data for K_s. Having access to these additional soils without unsaturated conductivity data is beneficial when conducting simulations to explore the effect of soil property variability within a soil class, for example. The moderate increase in error from about 0.705 to 0.930 is often an acceptable tradeoff to be able to work without unsaturated conductivity data, especially in variably-saturated analyses where the unsaturated flow rates are less significant.

Another consideration when using the MVGM SWCC is that the VGM model "has evolved to a de facto standard" (Vereecken et al. 2010); meaning that computing different parameter estimates is cumbersome. The parameter most affected by varying h_e is α, which increases on average by less than 2% when using $h_e = -0.1$ cm instead of $h_e = 0$ (VGM), and the other parameters change by less than 0.1%. However when using $h_e = -2$ cm instead of VGM, the average α for this dataset more than doubles in magnitude and the θ values are affected more significantly (around 1%), making refitting of the SWCC necessary. The goodness of fit, simplicity, and correspondence with VGM parameters make the use of an immutable estimate of $h_e = -0.1$ cm an attractive option in the absence of further information. Table 3 displays the mean and interquartile range for the saturated conductivity data and the fitted parameter estimates in the filtered UNSODA database using MVGM with $h_e = -0.1$ cm.

Confirmation of Filtering Criteria

This study used an outlier criterion based on K_s to remove samples suspected of having substantial macropore influence. While the soils in Fig. 1 display strong evidence of substantial macropore influence, the same is not true of all removed soils (any cutoff criteria would unfortunately remove some matrix-dominated soils and retain some macropore-dominated soils). Therefore a comparison of the MVGM ($h_e = -0.1$ cm) mean fits was made between the filtered dataset and the removed soils to test for a difference in macropore influence between the two groups. The MVGM model overpredicts the unsaturated conductivity more in macropore-dominated soils than in matrix-dominated soils when K is normalized to K_s [Eq. (4)]. Therefore, observing greater overprediction in the removed soils than in the filtered dataset would indicate greater macropore influence in the soils that were removed. To account for the influence of n on the fits (because the distributions of n might not be identical between the two groups), an analysis of covariance was performed to test for the difference between the filtered and removed soil groups while controlling for n. To linearize the analysis of covariance model and therefore meet the model assumptions

Table 3. Statistics of MVGM Model Parameters from Filtered UNSODA Dataset

		K_s [a]	θ_s	θ_r	$1/\alpha$	n
Texture	Sample size	(cm/d) (geometric)	(cm/cm) (arithmetic)	(cm/cm) (arithmetic)	(cm) (geometric)	(-) (arithmetic)
Clay	15	6.4 (1.6, 51.4)	0.51 (0.47, 0.55)	0.042 (0, 0.022)	34.8 (17.2, 112.6)	1.11 (1.07, 1.14)
Clay loam	4	6.6 (4.4, 18.7)	0.43 (0.43, 0.46)	0.066 (0, 0.125)	76.3 (18.5, 405.8)	1.18 (1.1, 1.2)
Loam	24	23.2 (10.9, 68.2)	0.46 (0.39, 0.51)	0.033 (0, 0.002)	24.8 (8.4, 69.3)	1.28 (1.12, 1.21)
Loamy sand	29	159.3 (60.4, 499)	0.39 (0.37, 0.41)	0.052 (0.03, 0.073)	24.2 (12.5, 42.4)	2.05 (1.51, 2.39)
Sand	66	186.9 (86.7, 536.7)	0.35 (0.32, 0.38)	0.046 (0.03, 0.07)	33.3 (22.3, 49.1)	3.12 (1.82, 4.01)
Sandy clay	2	0.1 (-)	0.39 (-)	0.264 (-)	1042.3 (-)	2.32 (-)
Sandy clay loam	16	3.5 (0.6, 21.7)	0.39 (0.35, 0.41)	0.084 (0, 0.193)	28.5 (11.8, 91.4)	1.27 (1.08, 1.2)
Sandy loam	52	42.7 (18.2, 120.1)	0.39 (0.34, 0.45)	0.046 (0, 0.076)	36.8 (19.2, 69.4)	1.77 (1.2, 1.49)
Silt	2	55.8 (-)	0.42 (-)	0.037 (-)	238.3 (-)	1.63 (-)
Silt loam	62	19.6 (7.5, 58.6)	0.45 (0.4, 0.48)	0.039 (0, 0.057)	79.8 (43.4, 175)	1.46 (1.19, 1.47)
Silty clay	8	10.2 (3.1, 69.6)	0.49 (0.44, 0.54)	0.183 (0.01, 0.29)	93.8 (54.3, 213.3)	1.41 (1.19, 1.68)
Silty clay loam	16	17.8 (2.1, 92.8)	0.54 (0.5, 0.59)	0.147 (0, 0.209)	40.8 (27.2, 85.6)	1.36 (1.15, 1.54)

Note: MVGM with $h_e = -0.1$ cm for the UNSODA dataset filtered to remove K_s above the cutoff values in Table 1 that are indicative of macropore influence. Values in parentheses are the 25 and 75% values. Means are computed either geometrically or arithmetically as specified.
[a]K_s values are not fitted, but are measurements from the soils selected from the UNSODA dataset.

Table 4. Data Points Removed from the UNSODA Database Prior to Model Fitting

Soil identifier	Removed data point	Reason for removal
1460	th = 0.73 when suction = 32 cm	Assumed data entry error; probably should have been 0.073 based on the values on either side, but the data point was deleted rather than guessing
2712	Porosity = 0.915	Too high for a porosity value, confirmed by the water content at 1 cm suction equal to 0.42
3091	Entire soil	The smallest suction measurement on the water retention curve is 27,000 cm, therefore proper fitting of the near-saturation water retention curve is not possible
4650	th_sat = 0.586	Does not match the porosity (0.38) or the value in the h-th table (0.38)
1383, 2611, 3101, 3102, 3110, 3111, 3112, 3243, 3371	Porosity	These porosity values were removed because they are substantially lower than the unsaturated water content data; assuming that they are equal to the saturated water content would cause the model fit to miss the unsaturated data

by normalizing and removing trends in the model residuals, the covariate was transformed to $1/n^3$ rather than n. This test shows that on average, the removed soils have a mean $\log_{10}(K)$ residual that is greater in magnitude than the filtered dataset by 0.52, with a 95% confidence interval of (0.16, 0.88). This increase in residual indicates that on average the MVGM model overpredicts the HCC in the removed soils by half an order of magnitude more than in the filtered dataset, showing that the group of removed soils is indeed different from the filtered dataset. The significance of this test was confirmed by a Kolmogorov-Smirnov test and a two-sample permutation test of means, both of which test for equivalence of the sampling distributions without requiring any assumptions about the sampling distribution as in the analysis of covariance. These nonparametric tests confirm that the two sampling distributions are not equivalent (p-values of 0.038 and 0.004, respectively). While these tests are insufficient to prove causality, the results are consistent with the hypothesis that the removed soils contain significantly more macropore influence than the retained soils.

Conclusions

A method was outlined for obtaining soil matrix hydraulic properties that may be used for variably-saturated hydrologic analysis such as those used to analyze watersheds or stormwater controls. The soils are first filtered based on Rawls et al. (1998) using the K_s upper limits in Table 1 to remove high-K_s soils on the assumption that they represent macropore dominance. It was shown that K_s may be used as the normalization constant for the MVGM model [Eq. (4)] once the high-K_s soils are removed. In the absence of unsaturated hydraulic conductivity data to fully calibrate the MVGM model, the MVGM model parameter h_e may be considered an immutable constant. These methods were applied on the UNSODA database where the immutable-h_e method was calibrated to $h_e = -0.1$ cm. A table of mean and interquartile ranges of MVGM parameter estimates (Table 3) was generated using the filtered UNSODA dataset with $h_e = -0.1$ cm; these results provide SHP estimates in the absence of specific data and highlight the substantial within-class and between-class variation of SHP.

The filtering procedure removes macropore-dominated samples from consideration leaving matrix-dominated soils, thereby providing engineers with a conservative estimate of soil properties when soil macropore structures cannot be estimated, as is often the case in engineering analyses. While the authors believe this K_s filter is an appropriate technique for practical implementation, more research is recommended for improved methods of quantifying the influence of macropores on the hydraulic conductivity of soils. The filtering process presented here is an approximate method that does remove a large portion of macropore-dominated soils, which is a conservative design approach. The result of this filtering was evaluated on the UNSODA database; the removed soils were statistically different from the retained soils in a manner consistent with macropore influence. Additionally, the removal of high-K_s soils allowed the MVGM model HCC to be normalized to K_s, making it applicable in variably-saturated simulations, unlike many typical VGM models, which are normalized to fit the unsaturated conductivity data instead. Finally, it was shown that the MVGM parameter h_e may be considered immutable at -0.1 cm for all soils in the absence of additional information to calibrate it. This value of h_e has several benefits: it substantially improves HCC predictions for low-n soils over the VGM model; it allows for the analysis of soils that do not have unsaturated conductivity data to fit the HCC; and being a small value, it allows for the use of VGM SWCC fit parameters if they are already known, rather than refitting the SWCC.

While it is possible to obtain SHP models with lower RMS residuals than with the immutable-h_e method (e.g., Schaap and van Genuchten 2006; Weynants et al. 2009), those more-precise models rely on additional fitting parameters for the HCC, resulting in a need for unsaturated HCC data, generally resulting in fewer available soil models. The more precise SHP fitting methods may certainly be used when data is available to calibrate them, however it is still recommended to first consider filtering the data to remove macropore-dominated soils when that is appropriate for the engineering analysis under consideration. If unsaturated HCC data are unavailable, then the SHP model presented in this paper using MVGM with immutable h_e may be used for reasonable estimates of SHP, however it should be remembered that the recommendation for $h_e = -0.1$ cm is not verified on soils above the K_s upper limit (Table 1), nor is it verified for extremely low conductivities (less than 0.01 cm/d). The parameter estimates shown in Table 3 may be used for estimating the soil matrix properties when specific soil data are not available. While Table 3 provides an estimate of the range of expected parameters (assuming that the filtered UNSODA database represents a random sample of these soil types), using the individual soil properties rather than a class-average table is still recommended when appropriate and possible.

The methods described in this article represent an advancement in practical methods for estimating soil matrix hydraulic properties for water resources engineering use. The K_s-filtering procedure is a step toward extracting soil matrix hydraulic property data for conservative engineering analysis. Additionally, this study is the first to provide large-scale quantitative evidence for an optimum value of h_e when that parameter must be considered immutable rather than fitted to an individual soil's measurements. The MVGM with $h_e = -0.1$ cm represents a predictive hydraulic conductivity model that improves on the standard VGM model when unsaturated conductivity data are unavailable for model fitting. The immutable-h_e method is a simple, yet significant improvement over the VGM model, which improves on hydrologic simulations that are presently implemented with VGM. If VGM model parameters are known along with K_s, these methods can be applied directly in the SWAP Richards equation solver, which already accepts MVGM model inputs. Therefore this study makes the MVGM model more accessible to engineers, who are typically unable to estimate MVGM model parameters and are instead forced to use less-accurate models (e.g., Weynants et al. 2009).

Appendix. Modifications to UNSODA Database

The data points listed in Table 4 were removed from the UNSODA database prior to model-fitting for the reasons described therein.

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Notation

The following symbols are used in this paper:

h = pressure head, negative in suction (L);
h_e = parameter affecting the hydraulic conductivity curve in the modified van Genuchten-Mualem model (L);
K = hydraulic conductivity (L/T);
K_0 = hydraulic conductivity curve normalization constant (L/T);
K_r = relative hydraulic conductivity (−);
K_s = saturated hydraulic conductivity (L/T);
m = van Genuchten-Mualem model parameter (typically equal to $1 - 1/n$) (−);
n = van Genuchten-Mualem model parameter (−);
S_c = effective degree of saturation when $h = h_e$ (−);
S_e = effective degree of saturation (−);
α = van Genuchten-Mualem model parameter (1/L);
τ = van Genuchten-Mualem model parameter (−);
θ = soil moisture content (L/L);
θ_r = residual soil moisture content (van Genuchten-Mualem model parameter) (L/L); and
θ_s = saturated soil moisture content (van Genuchten-Mualem model parameter) (L/L).

References

Børgesen, C. D., Jacobsen, O. H., Hansen, S., and Schaap, M. G. (2006). “Soil hydraulic properties near saturation, an improved conductivity model.” *J. Hydrol.*, 324(1–4), 40–50.
Coppola, A., et al. (2014). “Mapping solute deep percolation fluxes at regional scale by integrating a process-based vadose zone model in a Monte Carlo approach.” *Soil Sci. Plant Nutr.*, 60(1), 71–91.
Fredlund, D. G. (2006). “Unsaturated soil mechanics in engineering practice.” *J. Geotech. Geoenviron. Eng.*, 10.1061/(ASCE)1090-0241(2006)132:3(286), 286–321.
Gutmann, E. D., and Small, E. E. (2005). “The effect of soil hydraulic properties vs. soil texture in land surface models.” *Geophys. Res. Lett.*, 32(2), L02402.
Hohenbrink, T. L., and Lischeid, G. (2014). “Texture-depending performance of an in situ method assessing deep seepage.” *J. Hydrol.*, 511, 61–71.
Ippisch, O., Vogel, H.-J., and Bastian, P. (2006). “Validity limits for the van Genuchten-Maulem model and implications for parameter estimation and numerical simulation.” *Adv. Water Resour.*, 29(12), 1780–1789.
Jarvis, N. J. (2007). “A review of non-equilibrium water flow and solute transport in soil macropores: Principles, controlling factors and consequences for water quality.” *Eur. J. Soil Sci.*, 58(3), 523–546.
Kroes, J. G., Van Dam, J. C., Groenendijk, P., Hendriks, R. F. A., and Jacobs, C. M. J. (2009). “SWAP version 3.2: Theory description and user manual.” *Alterra Rep.*, Wageningen, Netherlands, 1649(02).
Leij, F. J., Alves, W. J., van Genuchten, M. Th., and Williams, J. R. (1996). “Unsaturated soil hydraulic database, UNSODA 1.0 user’s manual.” *Rep. EPA/600/R96/095*. U.S. Environmental Protection Agency, Ada, OK, 103.
Loosvelt, L., Pauwels, V. R., Cornelis, W. M., De Lannoy, G. J. M., and Verhoest, N. E. C. (2011). “Impact of soil hydraulic parameter uncertainty on soil moisture modeling.” *Water Resour. Res.*, 47(3), W03505.
MATLAB [Computer software]. MathWorks, Natick, MA.
Minasny, B., Hopmans, J. W., Harter, T., Eching, S. O., Tuli, A., and Denton, M. A. (2004). “Neural networks prediction of soil hydraulic functions for alluvial soils using multistep outflow data.” *Soil Sci. Soc. Am. J.*, 68(2), 417–429.
Mualem, Y. (1976). “A new model for predicting the hydraulic conductivity of unsaturated porous media.” *Water Resour. Res.*, 12(3), 513–522.
Nemes, A., Schaap, M. G., Leij, F. J., and Wösten, J. H. M. (2001). “Description of the unsaturated soil hydraulic database UNSODA version 2.0.” *J. Hydrol.*, 251(3–4), 151–162.
Pitt, R., Chen, S., Clark, S. E., Swenson, J., and Ong, C. K. (2008). “Compaction’s impacts on urban storm-water infiltration.” *J. Irrig. Drain. Eng.*, 10.1061/(ASCE)0733-9437(2008)134:5(652), 652–658.
Rawls, W. J., Brakensiek, D. L., and Saxton, K. E. (1982). “Estimation of soil water properties.” *Trans. ASAE*, 25(5), 1316–1320.
Rawls, W. J., Gimenez, D., and Grossman, R. (1998). “Use of soil texture, bulk density, and slope of the water retention curve to predict saturated hydraulic conductivity.” *Trans. ASAE*, 41(4), 983–988.
Rocha, D., Abbasi, F., and Feyen, J. (2006). “Sensitivity analysis of soil hydraulic properties on subsurface water flow in furrows.” *J. Irrig. Drain. Eng.*, 10.1061/(ASCE)0733-9437(2006)132:4(418), 418–424.
Schaap, M. G., and Leij, F. J. (2000). “Improved prediction of unsaturated hydraulic conductivity with the Mualem-van Genuchten model.” *Soil Sci. Soc. Am. J.*, 64(3), 843–851.
Schaap, M. G., and van Genuchten, M. Th. (2006). “A modified Mualem-van Genuchten formulation for improved description of the hydraulic conductivity near saturation.” *Vadose Zone J.*, 5(1), 27–34.
Tietje, O., and Hennings, V. (1996). “Accuracy of the saturated hydraulic conductivity prediction by pedo-transfer functions compared to the variability within FAO textural classes.” *Geodermal*, 69(1–2), 71–84.
Twarakavi, N. K. C., Sakai, M., and Šimůnek, J. (2009). “An objective analysis of the dynamic nature of field capacity.” *Water Resour. Res.*, 45(10), W10410.
Twarakavi, N. K. C., Šimůnek, J., and Schaap, M. G. (2010). “Can texture-based classification optimally classify soils with respect to soil hydraulics?.” *Water Resour. Res.*, 46(1), W01501.

Van Dam, J. C., and Feddes, R. A. (2000). "Numerical simulation of infiltration, evaporation and shallow groundwater levels with the Richards equation." *J. Hydrol.*, 233(1–4), 72–85.

van Genuchten, M. Th. (1980). "A closed-form equation for predicting the hydraulic conductivity of unsaturated soils." *Soil Sci. Soc. Am. J.*, 44(5), 892–898.

Vereecken, H., Weynants, M., Javaux, M., Schaap, M. G., and van Genuchten, M. Th. (2010). "Using pedotransfer functions to estimate the van Genuchten-Mualem soil hydraulic properties: A review." *Vadose Zone J.*, 9(4), 795–820.

Vogel, T., van Genuchten, M. Th., and Cislerova, M. (2000). "Effect of the shape of the soil hydraulic functions near saturation of variably-saturated flow predictions." *Adv. Water Resour.*, 24(2), 133–144.

Weynants, M., Vereecken, H., and Javaux, M. (2009). "Revisiting vereecken pedotransfer functions: Introducting a closed-form hydraulic model." *Vadose Zone J.*, 8(1), 86–95.